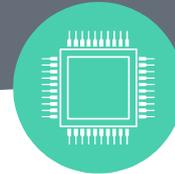


Diverse, Global and Amortised Counterfactual Explanations for Uncertainty Estimates



Overview

01 Counterfactual Latent Uncertainty Explanation (CLUE)

02 Why this paper?

03 δ -CLUE

04 Diversity Metrics

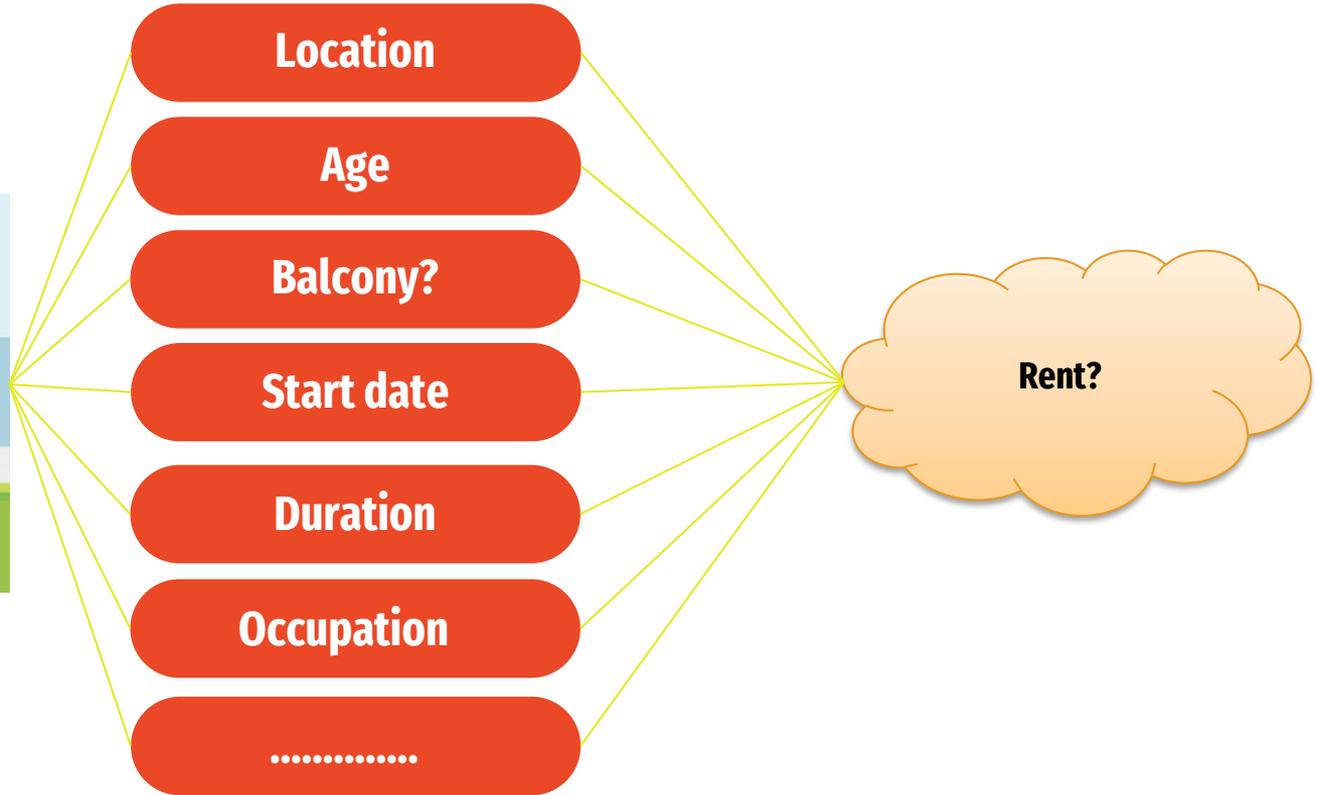
05 ∇ -CLUE

07 GLAM-CLUE

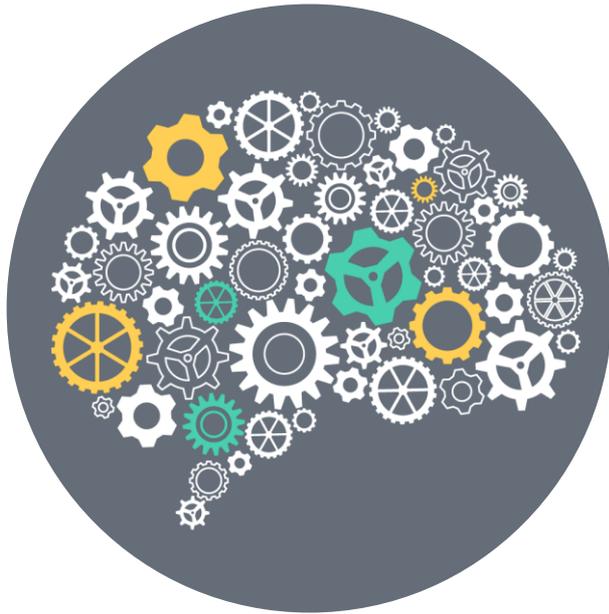
08 Performance Test

09 Future Work

Counterfactual Explanations



Counterfactual Latent Uncertainty Explanation (CLUE)



What is the smallest on-manifold change that can be done to an input so that our model becomes more certain

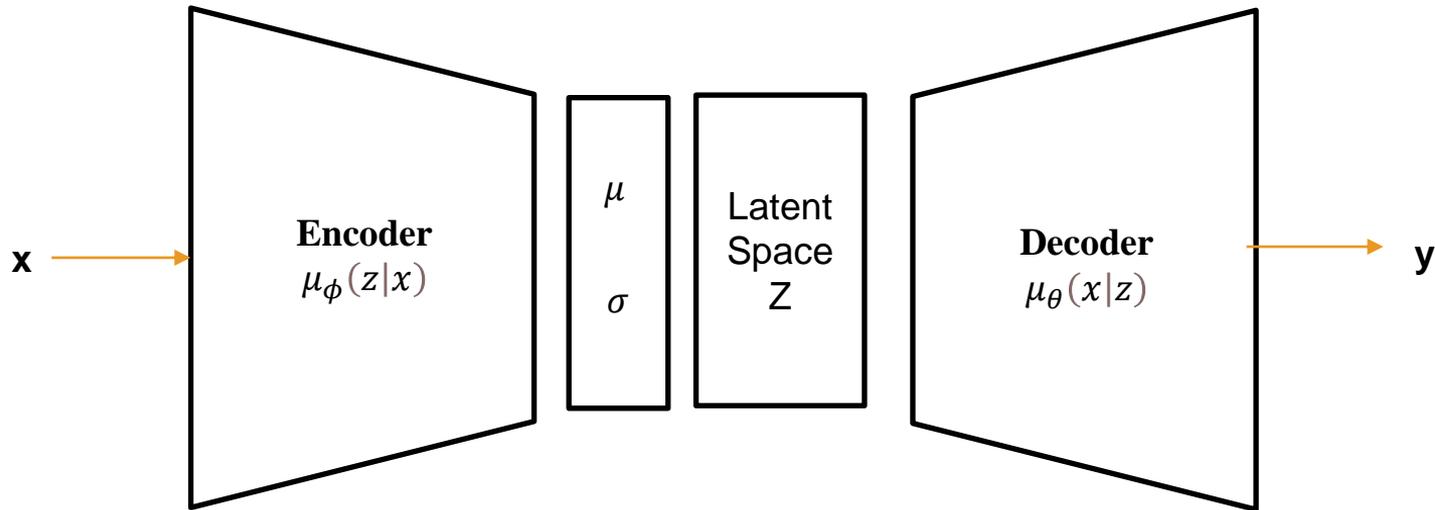
High Uncertainty

Incorrect prediction

More counterfactuals

Uncertainty explanations are a precedent for model explanation

CLUE Generation with Auxiliary Deep Generative Model with VAE



$$\mathcal{L}(\mathbf{z}) = \mathcal{H}(\mathbf{y}|\mu_{\theta}(\mathbf{x}|\mathbf{z})) + d(\mu_{\theta}(\mathbf{x}|\mathbf{z}), \mathbf{x}_0)$$

yields

$$\mathbf{x}_{\text{CLUE}} = \mu_{\theta}(\mathbf{x}|\mathbf{z}_{\text{CLUE}})$$

where

$$\mathbf{z}_{\text{CLUE}} = \arg \min_{\mathbf{z}} \mathcal{L}(\mathbf{z})$$

\mathcal{H} = Uncertainty estimate of a prediction \mathbf{y}

$$d(\mathbf{x}, \mathbf{x}_0) = \lambda_x d_x(\mathbf{x}, \mathbf{x}_0) + \lambda_y d_y(f(\mathbf{x}), f(\mathbf{x}_0)), \text{ where } f(\mathbf{x}) = \mathbf{y}$$

Why this paper?

Limitations of CLUE

Multiplicity Issue

δ -CLUE

Redundancy

∇ -CLUE (DIVERse CLUE)

Computational Inefficiency

GLAM-CLUE

δ -CLUE vs CLUE



δ -CLUE

Multiplicity is achieved by searching randomly in different areas of latent space

- Sampling around an input in latent space
- Gradient descent

Vs



CLUE also does this, but:

- Finds minima in a limited region of space
- Might stray far away from Counterfactuals

$$\mathbf{x}_{\delta\text{-CLUE}} = \mu_{\theta}(\mathbf{x}|\mathbf{z}_{\delta\text{-CLUE}}) \text{ where } \mathbf{z}_{\delta\text{-CLUE}} = \arg \min_{\mathbf{z}: \rho(\mathbf{z}, \mathbf{z}_0) \leq \delta} \mathcal{L}(\mathbf{z})$$

$$\mathbf{z}_0 = \mu_{\phi}(\mathbf{z}|\mathbf{x}_0)$$

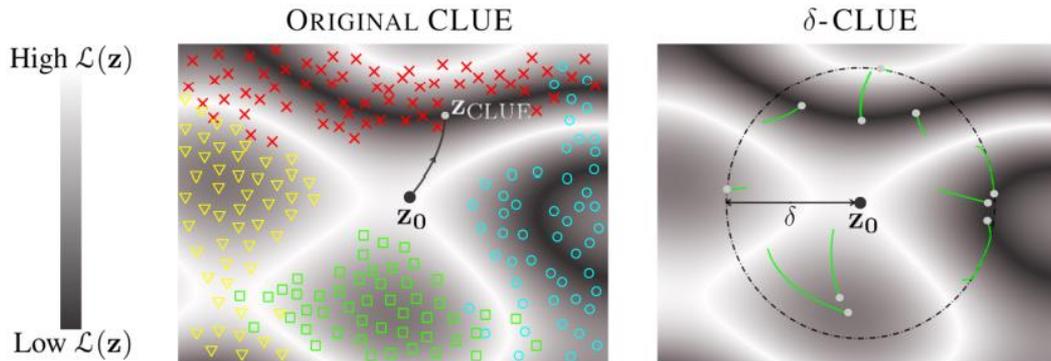
$$\rho(\mathbf{z}, \mathbf{z}_0) = \|\mathbf{z} - \mathbf{z}_0\|_2$$

Algorithm 3: δ -CLUE

Inputs: δ , k , \mathcal{S} , r , \mathbf{x}_0 , d , ρ , \mathcal{H} , μ_θ , μ_ϕ

```
1 Initialise  $\emptyset$  of CLUEs:  $X_{\text{CLUE}} = \{\}$ ;  
2 Set  $\delta$ -ball centre of  $\mathbf{z}_0 = \mu_\phi(\mathbf{z}|\mathbf{x}_0)$ ;  
3 for  $1 \leq i \leq k$  do  
4   Set initial value of  $\mathbf{z}_i = \mathcal{S}(\mathbf{z}_0, r, i, k)$ ;  
5   while loss  $\mathcal{L}$  has not converged do  
6     Decode:  $\mathbf{x} = \mu_\theta(\mathbf{x}|\mathbf{z}_i)$ ;  
7     Use predictor to obtain  $\mathcal{H}(y|\mathbf{x})$ ;  
8      $\mathcal{L} = \mathcal{H}(y|\mathbf{x}) + d(\mathbf{x}, \mathbf{x}_0)$ ;  
9     Update  $\mathbf{z}_i$  with  $\nabla_{\mathbf{z}} \mathcal{L}$ ;  
10    if  $\rho(\mathbf{z}_i, \mathbf{z}_0) > \delta$  then  
11      Project  $\mathbf{z}_i$  onto the surface of the  $\delta$ -ball as  $\mathbf{z}_i =$   
12         $\delta \times \frac{\mathbf{z}_i - \mathbf{z}_0}{\rho(\mathbf{z}_i, \mathbf{z}_0)}$ ;  
13    end if  
14  end while  
15  Decode explanation:  $\mathbf{x}_{\delta\text{-CLUE}} = \mu_\theta(\mathbf{x}|\mathbf{z}_i)$ ;  
16  if  $\mathcal{H}(y|\mathbf{x}_{\delta\text{-CLUE}}) < \mathcal{H}_{\text{threshold}}$  then  
17     $X_{\text{CLUE}} \leftarrow X_{\text{CLUE}} \cup \mathbf{x}_{\delta\text{-CLUE}}$ ;  
18  end if  
19 end for
```

Outputs: X_{CLUE} , a set of $n \leq k$ CLUEs



Different trials on δ -CLUE Algorithm

01

Range of δ values from 0.5 to 3.5

02

Two latent space loss functions:

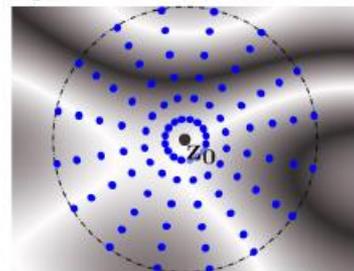
- Uncertainty: $\mathcal{L}_{\mathcal{H}} = \mathcal{H}$
- Distance: $\mathcal{L}_{\mathcal{H}+d} = \mathcal{H} + d$

03

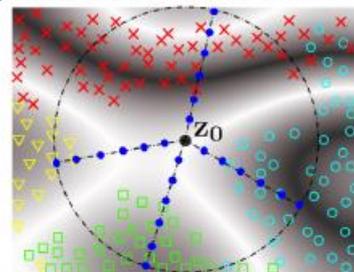
Two initialisation schemes like:

- Radially Uniform
- Nearest Neighbour

\mathcal{S}_1 : RADIALLY UNIFORM



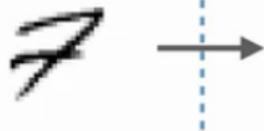
\mathcal{S}_2 : NEAREST NEIGHBOUR PATH



Low $\mathcal{L}(z)$

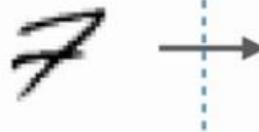
High $\mathcal{L}(z)$

INPUT X_0

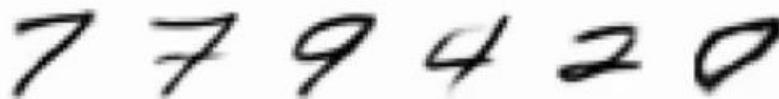


?

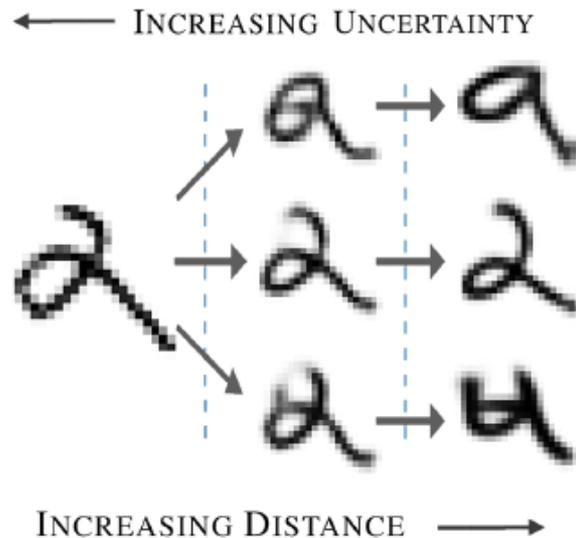
INPUT X_0



δ -CLUES



Uncertainty vs Distance Trade-off



The hyperparameters (λ_x, λ_y) controls this trade-off

Diversity Metrics (D)

01

Determinantal Point Process

$$\det(\mathbf{K}) \text{ where } \mathbf{K}_{i,j} = \frac{1}{1 + d(\mathbf{x}_i, \mathbf{x}_j)}$$

02

Average Pairwise Distance

$$\frac{1}{\binom{k}{2}} \sum_{i=1}^{k-1} \sum_{j=i+1}^k d(\mathbf{x}_i, \mathbf{x}_j)$$

03

Coverage

$$\frac{1}{d'} \sum_{i=1}^{d'} (\max_j (\mathbf{x}_j - \mathbf{x}_0)_i + \max_j (\mathbf{x}_0 - \mathbf{x}_j)_i)$$

04

Prediction Coverage

$$\frac{1}{c'} \sum_{i=1}^{c'} \max_j [(y_j)_i]$$

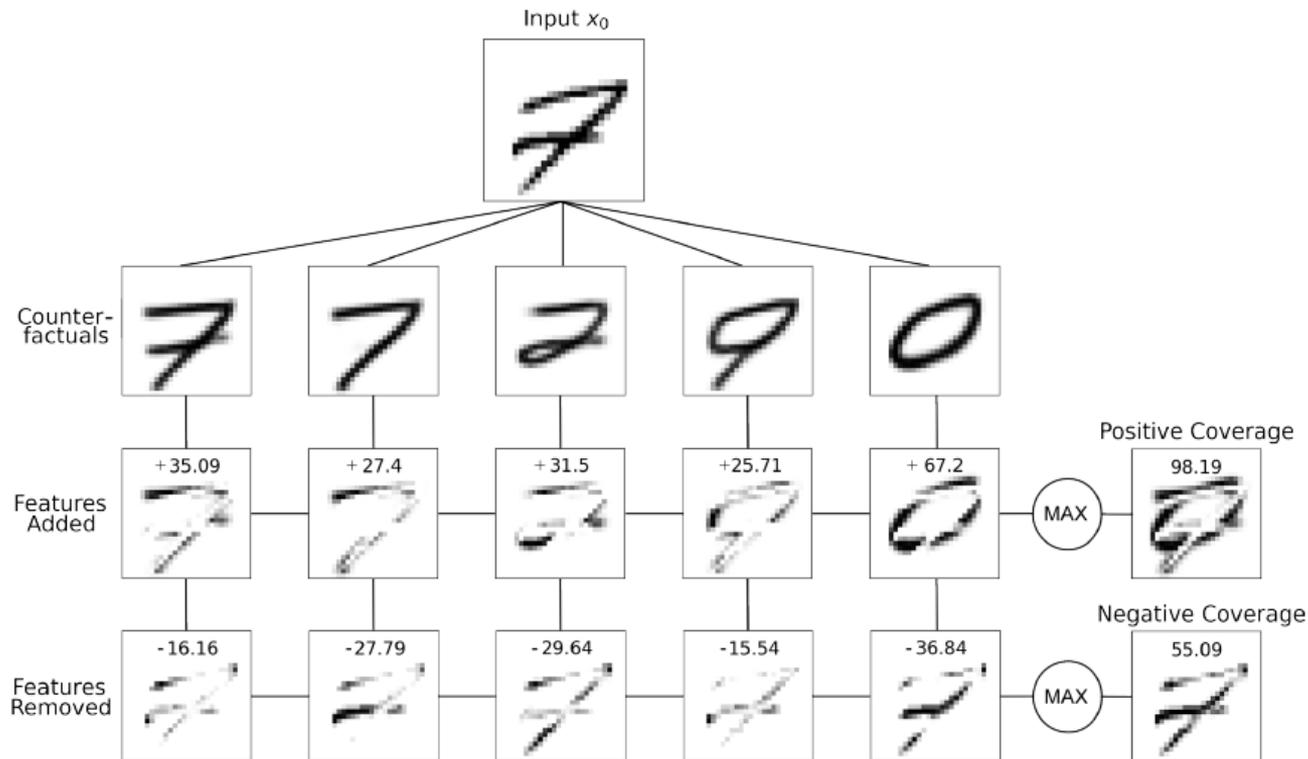
05

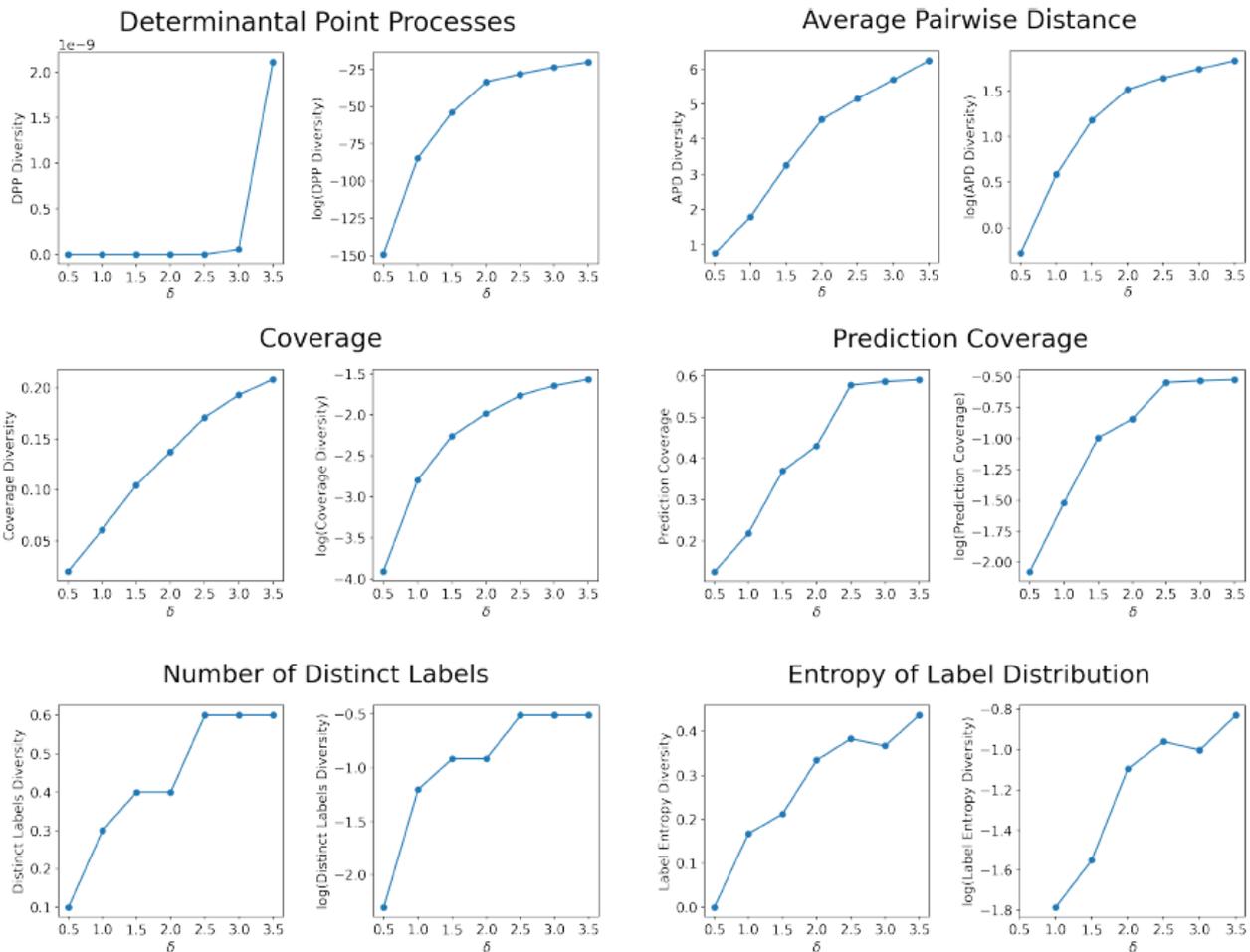
Distinct Labels/ Entropy of Labels

$$-\frac{1}{\log c'} \sum_{j=1}^{c'} p_j(k) \log p_j(k)$$



Coverage as a Metric





Diversity Optimization : ∇ -CLUE

01

Select the
Diversity Metric

02

Optimize k
counterfactuals

Simultaneous Diversity Optimization

$$\mathcal{L}(\mathbf{z}_1, \dots, \mathbf{z}_k) = -\lambda_D D(\mathbf{z}_1, \dots, \mathbf{z}_k) + \frac{1}{k} \sum_{i=1}^k \mathcal{L}(\mathbf{z}_i)$$

$$\mathcal{L}(\mathbf{z}_i) = \mathcal{H}(y|\mu_\theta(\mathbf{x}|\mathbf{z}_i)) + d(\mu_\theta(\mathbf{x}|\mathbf{z}_i), \mathbf{x}_0)$$

$$X_{\text{CLUE}} = \mu_\theta(X|Z_{\text{CLUE}})$$

$$Z_{\text{CLUE}} = \arg \min_{\mathbf{z}_1, \dots, \mathbf{z}_k} \mathcal{L}(\mathbf{z}_1, \dots, \mathbf{z}_k)$$

Sequential Diversity Optimization

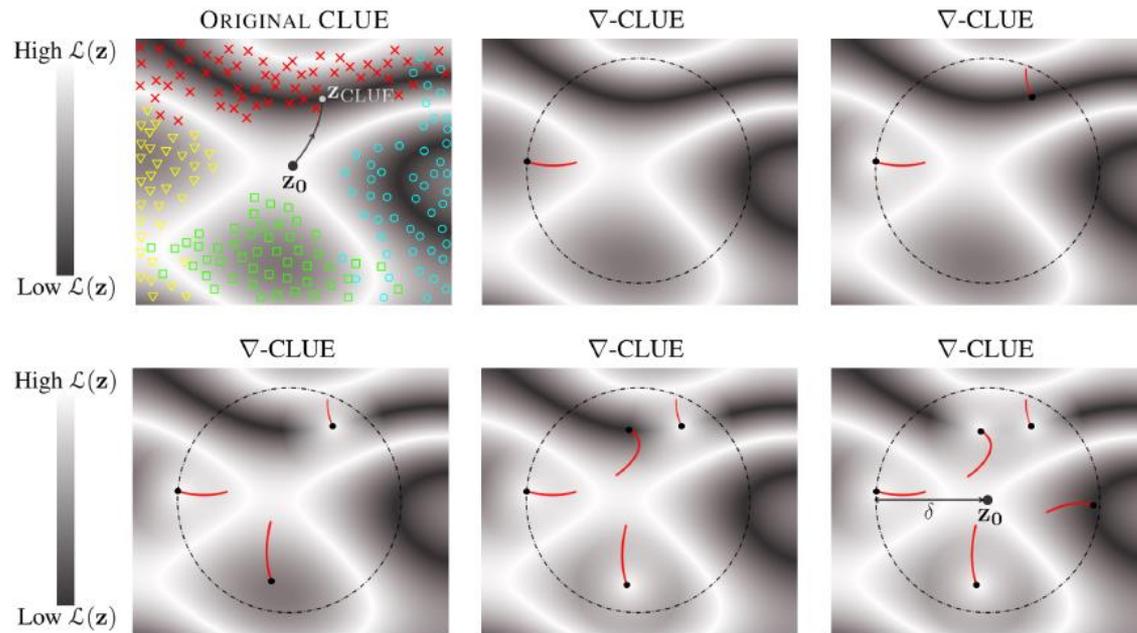
$$\mathcal{L}(\mathbf{z}) = \lambda_D D(Z_{\text{CLUE}} \cup \mathbf{z}) + \mathcal{H}(y|\mu_\theta(\mathbf{x}|\mathbf{z})) + d(\mu_\theta(\mathbf{x}|\mathbf{z}), \mathbf{x}_0)$$

Algorithm 1: ∇ -CLUE (simultaneous)

Inputs: $\delta, k, \mathcal{S}, r, \mathbf{x}_0, d, \rho, \mathcal{H}, \mu_\theta, \mu_\phi, D, \lambda_D$

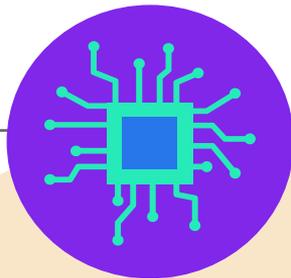
```
1 Initialise  $\emptyset$  of CLUES:  $X_{\text{CLUE}} = \{\}$ ;  
2 Set  $\delta$ -ball centre of  $\mathbf{z}_0 = \mu_\phi(\mathbf{z}|\mathbf{x}_0)$ ;  
3 for  $1 \leq i \leq k$  do  
4   Set initial value of  $\mathbf{z}_i = \mathcal{S}(\mathbf{z}_0, r, i, k)$ ;  
5 end for  
6 while loss  $\mathcal{L}$  has not converged do  
7   for  $1 \leq i \leq k$  do  
8     Decode:  $\mathbf{x}_i = \mu_\theta(\mathbf{x}|\mathbf{z}_i)$ ;  
9     Use predictor to obtain  $\mathcal{H}(\mathbf{y}|\mathbf{x}_i)$ ;  
10     $\mathcal{L}(\mathbf{z}_i) = \mathcal{H}(\mathbf{y}|\mathbf{x}_i) + d(\mathbf{x}_i, \mathbf{x}_0)$ ;  
11  end for  
12   $\mathcal{L}(\mathbf{z}_1, \dots, \mathbf{z}_k) = -\lambda_D D(\mathbf{z}_1, \dots, \mathbf{z}_k) + \frac{1}{k} \sum_{i=1}^k \mathcal{L}(\mathbf{z}_i)$ ;  
13  Update  $\mathbf{z}_1, \dots, \mathbf{z}_k$  with  $\nabla_{\mathbf{z}_1, \dots, \mathbf{z}_k} \mathcal{L}(\mathbf{z}_1, \dots, \mathbf{z}_k)$ ;  
14  for  $1 \leq i \leq k$  do  
15    Constrain  $\mathbf{z}_i$  to  $\delta$  ball using  $\rho(\mathbf{z}_i, \mathbf{z}_0)$ ;  
16  end for  
17 end while  
18 for  $1 \leq i \leq k$  do  
19   Decode explanation:  $\mathbf{x}_i = \mu_\theta(\mathbf{x}|\mathbf{z}_i)$ ;  
20   if  $\mathcal{H}(\mathbf{y}|\mathbf{x}_i) < \mathcal{H}_{\text{threshold}}$  then  
21      $X_{\text{CLUE}} \leftarrow X_{\text{CLUE}} \cup \mathbf{x}_i$ ;  
22   end if  
23 end for
```

Outputs: X_{CLUE} , a set of $n \leq k$ diverse CLUES



Sequential ∇ -CLUE Optimization

Global AMortised CLUE (GLAM-CLUE)



Input data

Finite set of Counterfactuals

Training Step

A mapper which learn global properties of uncertainty in latent space

$$\mathbf{z}_{\text{certain}} = G(\mathbf{z}_{\text{uncertain}})$$

Inference Step

Apply this mapper to test data to see the effect on uncertain data

Output data

$$\mathcal{L}(\theta | Z_{\text{uncertain}}, X_{\text{certain}}) = \lambda_{\theta} \|\theta\|_1 + \frac{1}{|Z_{\text{uncertain}}|} \sum_{\mathbf{z} \in Z_{\text{uncertain}}} \min_{\mathbf{x} \in X_{\text{certain}}} \|\mu_{\theta}(\mathbf{z} + \theta) - \mathbf{x}\|_2^2 \quad (2)$$

Algorithm 2: GLAM-CLUE (Training Step)

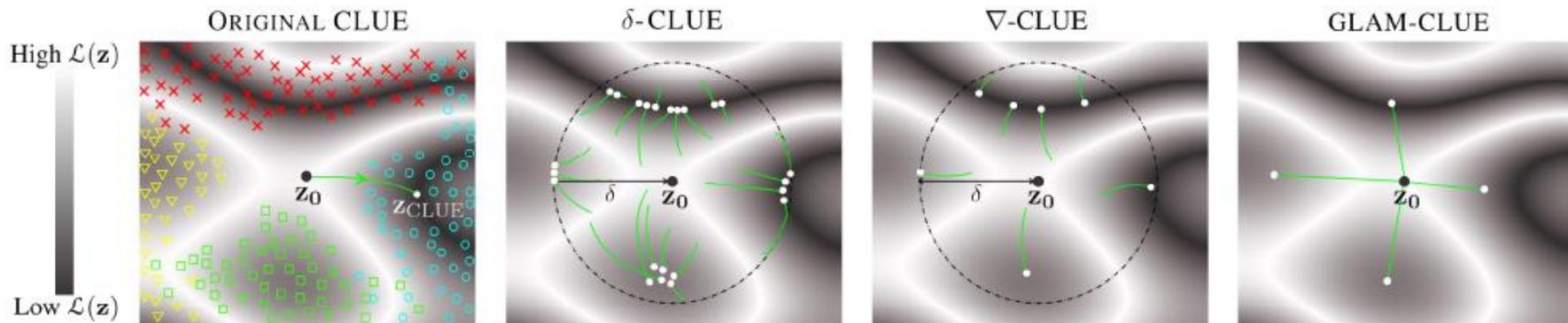
Inputs: Inputs $X_{\text{uncertain}}, X_{\text{certain}}$, groups $Y_{\text{uncertain}}, Y_{\text{certain}}$, DGM encoder μ_ϕ , loss \mathcal{L} , trainable parameters θ

- 1 for all groups $(i \rightarrow j)$ in $(Y_{\text{uncertain}}, Y_{\text{certain}})$ do
- 2 Select X_i from $X_{\text{uncertain}}, Y_{\text{uncertain}}$;
- 3 Select X_j from $X_{\text{certain}}, Y_{\text{certain}}$;
- 4 Encode: $Z_i = \mu_\phi(Z|X_i)$;
- 5 while loss \mathcal{L} has not converged do
- 6 Update $\theta_{i \rightarrow j}$ with $\nabla_{\theta_{i \rightarrow j}} \mathcal{L}(\theta_{i \rightarrow j} | Z_i, X_j)$;
- 7 end while
- 8 end for

Outputs: A collection of mapping parameters $\theta_{i \rightarrow j}$ for given mappers $G_{i \rightarrow j}$ that take uncertain inputs from group i and produce nearby certain outputs in group j



$$\mathbf{z}_j = G_{i \rightarrow j}(\mathbf{z}_i) = \mathbf{z}_i + \theta_{i \rightarrow j}$$



Performance Test

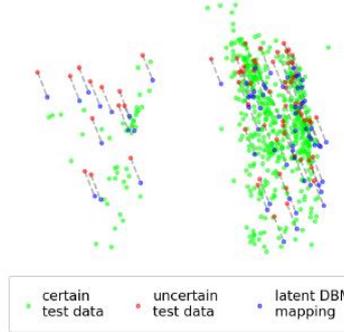
01 Difference Between Means (DBM)

Uncertain data to certain data in input or latent space

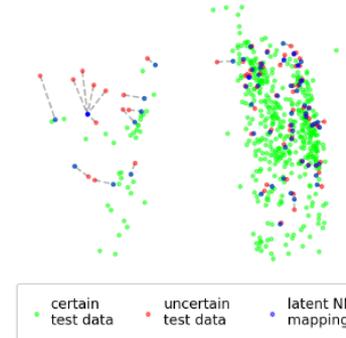
02 Nearest Neighbours (NN)

Used in high certainty training data in input or latent space

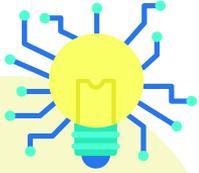
Latent DBM Mapping
On Unseen Test Data



Latent NN Mapping
On Unseen Test Data



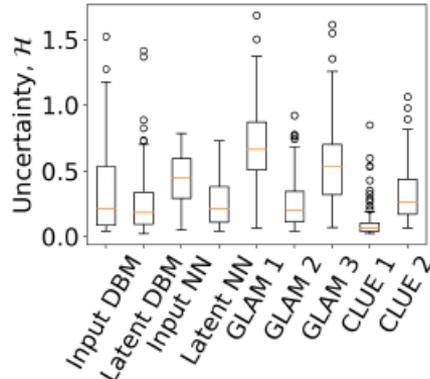
Performance Comparison



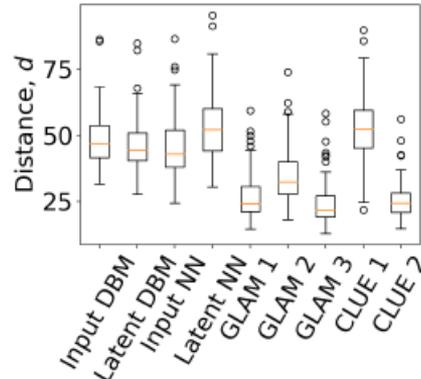
Input DBM	Latent DBM	Input NN
0.0306	0.0262	0.0236
Latent NN	GLAM-CLUE	CLUE
0.0245	0.0238	4.68

GLAM-CLUE outperforms these
baselines
.....almost 200 times faster

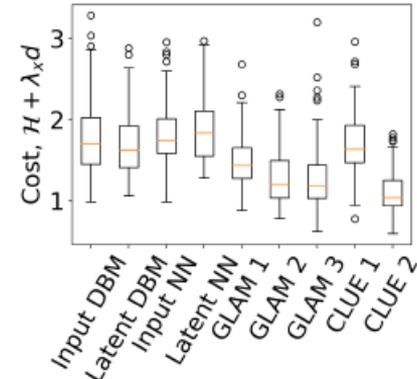
Uncertainties Comparison



Distances Comparison



Costs Comparison



Future Work

01

Data set dimensions

Using higher dimensional data set

02

Introduce different metric

Use FID scores to replace simple distance metric in evaluation and optimisation

03

Use different DGMs

Use DGM alternative like GANs instead of VAEs

Conclusion

- 01** Making CLUE more useful in practice
- 02** Proposed δ -CLUE and ∇ -CLUE to tackle the multiplicity and diversity issues
- 03** Introduced GLAM-CLUE which tackles the computational inefficiency caused on large data sets with δ -CLUE and ∇ -CLUE

References

- <https://towardsdatascience.com/understanding-variational-autoencoders-vaes-f70510919f73>
- Antorán, Javier, et al. "Getting a clue: A method for explaining uncertainty estimates." *arXiv preprint arXiv:2006.06848* (2020).
- Ley, Dan, Umang Bhatt, and Adrian Weller. " Δ -CLUE: Diverse Sets of Explanations for Uncertainty Estimates." *arXiv preprint arXiv:2104.06323* (2021).
- <https://slideslive.com/38955757/deltaclue-diverse-sets-of-explanations-for-uncertainty-estimates?ref=recommended>



THANK YOU

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